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Invited Article

On the Distribution Efficiency of an Optimal Monetary Policy

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ABSTRACT

The paper studies the impacts of an optimal monetary policy on the distribution and production efficiencies by using a framework of multiple types of household and assets. It extends the work of Xiang (2013) by adding a new type of risky asset, known as Lucas tree, into an existing money-bond model. Some new results can be generated by requiring that all asset markets must satisfy the non-arbitrage profit condition. For example, regardless of insufficient liquidity, a zero nominal interest rate as suggested by the Friedman rule becomes an optimal monetary policy that can lead the economy to its full distribution efficiency and also lower its production inefficiency at the same time.

Keywords: Distribution efficiency, optimal monetary policy, asset price, non-arbitrage profit, liquidity, nominal interest, Friedman rule

JEL Classification: E23, E31, E4, E5

1. Introduction

Bhattacharya, Haslag, and Martin (2005) argued that the Friedman rule[1] is optimal only in the case of a homogeneous agent model. The existence of heterogeneity among agents confirms the redistributive effect of monetary policy and turns the Friedman rule into a suboptimal policy. Andolfatto (2011) used a quasi-linear environment with competitive markets to study the distributive benefits of illiquid bonds under an endowment economy. Xiang (2013), by incorporating a productive sector in the model of Andolfatto (2011), analyzed the interaction of distribution and production efficiencies when those heterogeneous agents can use money-bond exchanges to cope with liquidity shocks.

This paper sketches a preliminary plan to integrate the current existing optimal monetary policy under heterogeneous agents into a setting of asset price function. The results of Xiang's (2013) model at the stationary equilibrium are (i) money has a lower return than an illiquid bond, (ii) the size of the return differential is higher in a high-inflation environment, and (iii) if consumers are sufficiently risk averse, then the distribution efficiency gains from using illiquid interest-bearing bonds to channel liquidity among agents will be higher than the production efficiency loss being generated by an inflationary monetary policy.

2. Equilibrium Allocation in a Multiple-Asset Model with Heterogeneous Households

Based on the model in Xiang (2013), there are two groups of heterogeneous households living in an economy that can produce only one type of output. Each household, *i*

[1] See Friedman (2005).

$\in$ [0, 1], is assigned to either group l or group h with equal chance for each period $t = 0, 1, 2, 3, \ldots, \infty$.

Each period t is divided into two sub-periods. During the first sub-period, all households live at the same location, named location 0. Their utility in the first sub-period is linear in x_t, where $x_t \in$ R denotes household consumption (or production if negative) in the first sub-period at date t. This first sub-period output x_t is assumed to be perishable and produced by using labor effort.

At each date t, a liquidity shock, ω_t, on consumer type is realized at the beginning of the second sub-period, where $\omega_t \in \{\omega_l = 1, \omega_h = \eta\}$ and $1 < \eta < \infty$. Such liquidity shock is assumed to be *i.i.d.* across consumers within each group and over time. During the second sub-period, a consumer derives utility, $\omega_t u(c_t)$ from consuming $c_t \in R_+$ units of second sub-period goods. Utility function in the second sub-period, $u(c_t)$, has a constant relative risk averse coefficient $\rho \equiv -\frac{-c_t u''(c_t)}{u'(c_t)} > 0$, where $u''(c_t) < 0, u'(c_t) > 0, \lim_{c \to 0} u'(c_t) = \infty$ and $\lim_{c \to \infty} u'(c_t) = 0$. Let $y_t \in R_+$ be the perishable output produced in the second sub-period. $g(y_t)$is a cost function with $g'(y_t) > 0$ and $g''(y_t) < 0$.

For any household $i \in$ [0, 1], the expected lifetime utility function is a quasi-linear function defined as

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[x_t(i) + \omega_t(i) u\big(c_t(i)\big) - g\big(y_t(i)\big)\right] \quad (1)$$

with a discount rate $\beta \in (0, 1)$.

During the second sub-period of each t, each household $i \in$ [0, 1] finds out about its household type when the idiosyncratic preference shocks is realized at the beginning of the second sub-period. Such realization of preference shocks is private information. Each household is composed of a consumer and a producer. After the consumer type is realized, all producers from a household type *l (h)* must sell their second sub-period output to consumers from a household type *h (l)*. In other words, households are not allowed to consume their own output produced in the second sub-period. Fiat money is introduced into the economy as a mean of exchange because individual transaction histories cannot be traced or monitored.

In this paper, a new risky asset, Lucas trees, from Lucas (1978) is added into the original model above. The number of trees, S, is equal to the number of consumers. It is assumed that trees cannot be used to purchase yield from tree, d_t, which is a random variable. The realization of d_t becomes known to all at the second sub-period of each t. It is assumed that the stochastic process of d_t follows a Markov process with a transition function $F(x', x) = \Pr(d_{t+1} \leq x' | d_t = x_t)$, where $F: R_+ x R_+ \rightarrow R$ is a continuous function. At the first sub-period of each t, household is assigned to own s trees from t to $t+1$, where $s>0$. During the first sub-period, each household of type j can sell s_j trees, $0 \leq s_j \leq S$, (which is negative for a purchase) in the asset market for money. Tree owner has the right to collect the non-storable fruit dividends, d_t at the first sub-period of each t. Let p_t denote the market price of tree during period t. Let ω_t and d_t be two independent random variables for all t.

The government issues money, M_t, and bonds, B_t. New bonds are sold at the first sub-period of each t at a present discount price $0 < \delta < 1$. All bonds will be redeemed at par for money on the next period. Bonds are riskless asset that can be converted into future money. It is assumed that bonds cannot be used to purchase goods, but can be traded for money at a competitive price, δ_2, in a secondary bond market that opens during the second sub-period. In this multiple asset model, money supply must satisfy $M_{t+1} = M_t + B_t - \delta B_{t+1} - (p_t + d_t)s_t$.

At the long-run stationary equilibrium, let money supply expand at a constant rate for all t, then

$$\mu = \frac{M_{t+1}}{M_t}, \tag{2}$$

The value of μ also reflects long-term inflation rate.

In the first sub-period of each t, households have $z_t \geq 0$ units of fiat money, and have $m_t \geq 0$ at the second sub-period. Denote real number by $a_t \equiv v_1 z_t$ at the first sub-period and by $q_t \equiv v_2 m_t$ at the second sub-period where v_1 and v_2 are the values of money in the first and second sub-periods, respectively, and define that $\phi \equiv (v_1 / v_2)$. Real money transfer and real money stock are $\tau_t \equiv v_1 T_t$ and $Q_t = v_2 M_t$, where $T_t \geq 0$ is a lump-sum transfer to household.

During the first sub-period, a household decides how much to consume and how much money, bonds and risky assets to take to the second sub-period. Let a denote total real balances, b denote real holdings of newly issued bonds, and $p_t s_t$ denote real holding of risky asset purchased by household in the first sub-period. Bonds will be redeemed at par for money on the next period. Bonds and risky asset cannot be used to purchase goods.

The households' problem in (1) can be solved for a long-run stationary equilibrium, by

$$W(a,d) \equiv \max_{q\geq 0, b\geq 0, s\geq 0} \{a - \phi(q + \delta_2 b + (p+d)(S - s_t)) + V(q,b,s)\}, \tag{3}$$

where $V(q, b, s)$ is the value of entering the second sub-period at each t with real money q, real bonds b and real risky assets *s*. It is also a weighted-average value of entering the second sub-period at each *t* of all household types, or

$$V(q,b,s_t) \equiv \alpha V_l(q,b,s_t) + (1-\alpha)V_h(q,b,s_t), \quad 0 < \alpha < 1 \tag{4}$$

The real money demand q, real bond demand b and demand for Lucas tree *s* are characterized by

$$\phi = \frac{\partial V(q,b,s)}{\partial q}, \tag{5}$$

$$\delta\phi = \frac{\partial V(q,b,s)}{\partial b}, \tag{6}$$

$$(p+d)\phi = \frac{\partial V(q,b,s)}{\partial s}, \tag{7}$$

and envelope theorem $W'(a,d) = 1$.

In the second sub-period when the household type $j \in \{l, h\}$ is realized, household type *j* solves the following problem for a long-run stationary equilibrium,

$$V_j(q,b,s) \equiv \max_{b_j,s_j,c_j,y_j} \{ \omega_j u(c_j) - g(y_j) + \beta W(a_j^+, d) + \xi_j(b - b_{jt}) + \gamma_j (p+d)(S - s_j) + \lambda_j (q + \delta_2 b_j + p_t s_j - c_j)\}, \tag{8}$$

Given that

$$a_j^+ = \frac{\phi}{\mu}\left[(b - b_j) + (p+d)(S - s_j) + (q + \delta_2 b_j + p s_j - c_j) + y_j\right], \tag{9}$$

And

$$0 \le s_j \le S, \;\; j = l, h \tag{10}$$

$$\sum_j s_j = 0\,, \quad j = l, h \tag{11}$$

where a^+ denotes the real money balances taken into the next period. Let ξ_j, γ_j, and λ_j be Lagrange multipliers, and note that $W'(a, d) = 1$.

Then, the first-order conditions, for a long-run stationary equilibrium, are,

$$g'(y_j) = \frac{\beta\phi}{\mu}, \tag{12}$$

$$\lambda_j = \omega_j u'(c_j) - \frac{\beta\phi}{\mu}, \tag{13}$$

$$\xi_j = \delta_2 \omega_j u'(c_j) - \frac{\beta\phi}{\mu}, \tag{14}$$

$$(p + d)\gamma_j = \lambda_j p - \frac{\beta\phi}{\mu} d, \tag{15}$$

Substituting λ from (13) into (15),

$$\gamma_j = \left(\frac{p}{p+d}\right) \omega_j u'(c_j) - \frac{\beta\phi}{\mu}, \tag{16}$$

The envelope theorem gives $\frac{\partial V_j(q,b,s)}{\partial q} = \omega_j u'(c_j)$, $\frac{\partial V_j(q,b,s)}{\partial b} = \delta_2 \omega_j u'(c_j)$ and $\frac{\partial V_j(q,b,s)}{\partial s} = (p)\omega_j u'(c_j)$. Then, from equation (4), one can also have

$$\frac{\partial V(q,b,s)}{\partial q} = \alpha \frac{\partial V_j(q,b,s)}{\partial q_j} + (1-\alpha) \frac{\partial V_h(q,b,s)}{\partial q_h}, 0 < \alpha < 1, \tag{17}$$

Hence,

$$\frac{\partial V(q,b,s)}{\partial s} = \alpha\left(\frac{p}{p+d}\right)\omega_j u'(c_j) + (1-\alpha)\left(\frac{p}{p+d}\right)\omega_h u'(c_h), \tag{18}$$

Referring to (5), (6), the envelope theorem $\frac{\partial V_j(q,b,s)}{\partial q} = \omega_j u'(c_j)$, and $\frac{\partial V_j(q,b,s)}{\partial b} = \delta_2 \omega_j u'(c_j)$, one obtains $\delta_2 = \delta$. It means that the secondary market price for bonds must be the same as the issuing price.

Let type *l* households buy equity shares while type *h* households sell equity shares in the asset market. Type *l* consumers must satisfy a slack constraint, $\gamma_t = 0$, and thus equation (16) gives

$$\left[\frac{p}{p+d}\right] u'(c_j) = \frac{\beta\phi}{\mu}, \tag{19}$$

From (5), (17) and (19), one has

$$\left[\frac{p}{p+d}\right]\frac{\mu}{\beta} u'(c_j) = \alpha\omega_j u'(c_j) + (1-\alpha)\omega_h u'(c_h), \tag{20}$$

By combining the terms of $u'(c_j)$ in (20), one obtains the relationship of household's marginal utility, for a long-run stationary equilibrium as

$$\frac{\left[\frac{p}{p+d}\right]\mu - \alpha\beta}{(1-\alpha)\beta} u'(c_j) = \eta u'(c_h), \tag{21}$$

Consider (12) and (19), one obtains

$$g'(y) = \left[\frac{p}{p+d}\right] u'(c_j) = \delta U'(c_j), \quad (22)$$

Goods *y* market clearing condition requires that

$$c_j + c_h = 2y, \quad (23)$$

Money market clearing condition requires that

$$q = Q, \quad (24)$$

The market clearing conditions for the bond market at the first and second sub-periods are

$$b = \theta q, \quad (25)$$

$$b_j + b_h = 0, \quad (26)$$

The market clearing condition for the tree market at the first period is

$$s_j + s_h = 0, \quad (26)$$

Since type *h* household must be selling bonds, it must be that $b_h > 0$.

Then, the equilibrium allocation (c_l, c_h, y) is fully characterized by equations (21), (22) and (23), given any monetary policy (μ, θ).

In a monetary policy framework that has only money and interest-bearing bonds as studied by Kocherlakota (2005) and Xiang (2013), a policy authority may have at most three different monetary policy instruments, which are money supply μ, bonds to money ratio θ, and a secondary market price for bonds δ. However, if the model is allowed to have one more type of risky asset, one obtains, in the case of $\xi_j = \gamma_j = 0$, from equations (14) and (19), the following non-arbitrage condition

$$\delta = \left(\frac{p}{p+d}\right), \tag{27}$$

Equation (27) clearly states that the secondary market price for bonds must equal to an inverse of the market rate of return of trees. This condition is a result of the non-arbitrage profit condition that holds true for all asset markets when a rational expectation equilibrium exists. Thus, the number of feasible monetary policy instruments in this extended model of money, bonds and a risky asset is reduced to just two choices, which are μ and θ as compared to those previous models of money and bonds.

3. Feasible Optimal Monetary Policies in the Case of Multiple Assets

In order to see clearly the impact of monetary policy on the distributive efficiency under a multiple-asset model, one may start by exploring the characteristics of those monetary instruments, μ and θ at the stationary equilibrium. Let M_{t+1} define the next period money supply by

$$M_{t+1} = M_t + B_t - \delta B_{t+1}, \tag{28}$$

Equation (28) states that money supply in the next period must equal to the existing money supply plus the value of bonds that are redeemed, and then subtracting that result with the value of new bonds being issued in the next period.

By dividing both sides of equation (28) by M_t and rearranging the terms, one obtains at the stationary equilibrium,

$$\mu = \frac{1+\theta}{1+\delta\theta}, \tag{29}$$

Where $\mu \equiv \frac{M_{t+1}}{M_t}, \theta \equiv \frac{B_t}{M_t}, 0 < \delta \leq 1,$ and nominal interest rate is $i = \left(\frac{1}{\delta}\right) - 1 \geq 0$.

Note that the specific value of the term $\left(\frac{p}{p+d}\right)$ in equation (27) is given by the preference function of type j household, $\omega_t u(c_t)$. Let assume for simplicity that

$$\omega_t u(c_t) = \omega_t \ln(c_t), \tag{30}$$

Then, it can be shown that, at the stationary equilibrium, Lucas tree pricing function must be

$$p = \left(\frac{\beta}{1-\beta}\right) d, \;\; 0 < \beta < 1, \tag{31}$$

where β is the discount factor. Substituting equation (31) into (27) one obtains,

$$\delta = \frac{p}{p+d} = \beta, \tag{32}$$

where the nominal interest rate is $i = \left(\frac{1}{\delta}\right) - 1 > 0$.

It is also true that the market rate of return of Lucas tree at stationary equilibrium, Ω, must be

$$\Omega = \frac{p+d}{p} = \frac{1}{\beta} > 1, \tag{33}$$

Substituting the value of δ in (32) into (29), one can determine that the optimal money growth at the stationary equilibrium must be positive because

$$\mu = \left(\frac{1+\theta}{1+\beta\theta}\right) > 1, \tag{34}$$

The gross real interest rate according to Fisher equation as in Xiang (2013) is defined as

$$R \equiv \frac{1+i}{\mu} = \frac{1}{\delta\mu} \tag{35}$$

By using equations (32) and (34), it must be that

$$R = \frac{1+\beta\theta}{\beta(1+\theta)} \tag{35}$$

Equation (35) implies that

$$R < \frac{1}{\beta} \tag{36}$$

Equation (36) falls into the case which is called by Xiang (2013) as the 'insufficient liquidity' case. This is a case when $b_h = b$ $(s_h = s)$ and so $\xi_h = 0$ in equation (14) ($\gamma_h = 0$ in (16)). It implies that $\delta\eta u'(c_h) > \beta\frac{\phi}{\mu}$, or $\left(\left(\frac{p}{p+d}\right)\eta u'(c_h) > \beta\frac{\phi}{\mu}\right)$, or type h consumers cannot get as much liquidity from bond sales (risky asset sales) as they need.

In this case, the government cannot purchase more bonds from type h households since they don't have any bonds left. Government cannot increase δ because such action violates equation (27). The only policy instrument available is to print more money to circulate in the economy. Such an action will certainly increase inflation rate μ and end up with higher productive inefficiency.

4. Higher Inflation Intensifies Both Distribution and Production Inefficiencies

The distribution efficiency in the economy with heterogeneous household is defined by

$$D \equiv \frac{u'(c_j)}{\eta u'(c_h)} \tag{37}$$

Full distribution efficiency can be obtained only when $D = 1$. Distribution efficiency rises (falls) with D when $D < 1$ ($D > 1$) because a shift of a marginal unit of consumption from a type *l* (type *h*) household to a type *h* (type *l*) household can increase total welfare.

The production efficiency is defined by

$$P \equiv \frac{g'(y)}{max\{u'(c_j), \eta u'(c_h)\}} \tag{38}$$

Production efficiency is measured by marginal comparison of production cost and utility gains of agents who value consumption the highest.

By substituting equation (32) into (21), one obtains from (37)

$$D \equiv \frac{\beta(1-\alpha)}{\beta(\mu-1)} = \frac{1-\alpha}{\mu-\alpha} \leq 1 \tag{39}$$

The optimal monetary policy, in terms of distribution efficiency $D = 1$, requires that $\mu = 1$. This optimal policy is in line with the zero nominal interest rate as indicated by the

Friedman rule. Equation (39) clearly states that inflation is bad for distribution efficiency, $D < 1$.

Only type h household is restricted by liquidity constraint, so $\eta u'(c_h) > u'(c_j)$ and thus, equation (38) becomes

$$P \equiv \frac{g'(y)}{\eta u'(c_h)} \tag{40}$$

By using equation (20) and (21), one can rewrite equation (40) as

$$P = \delta \frac{(1-\alpha)}{(\mu-\alpha)} = \delta D < 1, \tag{41}$$

Production inefficiency (P < 1) occurs even in the period when Friedman rule, $\mu = 1$, is implemented.

5. Conclusion

The stationary equilibrium in this extended model requires that all asset markets must satisfy a non-arbitrage profit condition. As a result, the value of discounted bond price in the secondary market, δ, must be endogenously determined inside the model so that it is no longer a policy instrument as in the case of Xiang (2013).

Hence, under the situation of insufficient liquidity, in which real interest rate being lower than $1/\beta$, a full distribution efficiency level, D = 1, is still possible to reach providing that the nominal interest rate is set to zero as suggested by the Friedman rule. This result cannot be strictly guaranteed by the outcomes of Xiang (2013).

In addition, inflation clearly has deleterious effects on distribution and production efficiencies in the extended model.

The remaining challenges for future researches on this issue of optimal monetary policy is to explore the presence and implication of a speculative bubble in a non-stationary framework that may relate to the distribution and production efficiencies. This line of research has the potential to generate better understanding about the negative effect of an optimal monetary policy in the situation of asset price bubbles.

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